8.012 Review Guide

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General concepts:

1. Rotational coordinates, velocity, acceleration (p. 29, 31, 33, 38)
2. Newton's Laws
3. Conservation of Momentum
4. Conservation of Energy
5. Oscillations
6. Rotation
7. Angular Momentum and Torque
8. Angular Kinetic Energy
9. Fictitious Forces
10. Central Force Motion

Problem Solving Techniques:

1. Draw a picture of the system carefully.
2. Label all information-- forces, torques, velocities, angles, masses, etc.
3. Understand what the question is asking.
4. Use your potential problem solving techniques in order of difficulty:
5. Energy is the easiest because it's a scalar and has a conservation law.
6. Momentum is the next easiest because of conservation laws.
7. Forces are the hardest to solve with.

Problem class one-- Kinematics: use the equations of motion from chapter one.

Problem class two-- Newton's Laws:

1. Label clearly all forces acting on each part of your system. It takes some time to draw this for every part of your system, but it will get you the right answer.
2. Write out all the force equations for each part of your system. You should have one equation for each component.
3. Find correlations between variables not given by the force equations.
4. For systems of pulleys, imagine the string moving a distance x and find a relationship, for example, between x_1, x_2, and x_3. Differentiate with respect to time twice, and voila, now you have the same relationship for a_1, a_2, and a_3.

5. Solve the equations to get your answer.

Problem class three-- conservation of momentum, linear:

1. Conservation applies if there are no external forces. This must be used to solve collisions, energy conservation only applies in the very special case of an elastic collision.
2. The change in momentum divided by the change in time is the force in the limit of small delta. $\frac{dP}{dt}=F$
3. In order to solve problems with a stream of mass (i.e. a spray of sand into a train)--
   1. Write an equation representing the total momentum of the system at an arbitrary time t (not t=0). Especially with these "sand problems," make sure to include the momentum of both the "car" and the "delta m" of mass.
   2. Write an analogous equation representing the total momentum of the system at time $t+\Delta t$. Use the approximation that $\Delta m \Delta v \approx 0$. Use the relation that $v(t+\Delta t)=v+\Delta v$.
   3. Use conservation of momentum to write that $P(t+\Delta t)-P(t)=F\Delta t$. Elminate the differential mass by using $\Delta m=b\Delta t$ where $b=\frac{dm}{dt}$. You can then get the equation in terms of Δv and Δt and divide through by Δt to get $\lim \frac{\Delta v}{\Delta t}=\frac{dv}{dt}=acceleration$
   4. Use the acceleration and kinematics to solve for the future state of the objects in question.
4. If you are asked for how much force is required to pull things involved in collisions, remember that $P_{final}-P_{initial}=F_{average}\Delta t$. F is the average force over the interval Δt , so if you have, say, a guy grabbing a moving rope every 5 seconds, Δt would be 5 seconds.

Problem class four-- conservation of energy:

1. Conservation applies if there is no work done on the system by forces unaccounted by for by your equations. This means that with systems with friction or heat loss, you need to take the energy lost into account.
2. The change in energy is equal to the integral of the force with respect to the distance traveled in the direction of the force. $\Delta E=\int \vec{F}\cdot d\vec{r}$
3. Gravity-- $U=mgh$. The location of h=0 is arbitrary but needs to be consistent.
4. Kinetic-- $KE=\frac{1}{2}mv^2$ where v is the velocity of an object of mass m.

5. Spring-- $U=\frac{1}{2}kx^2$ where x is the distance displaced from equilibrium.
6. This is useful for solving problems involving falling things and finding final velocities of moving things and final heights. Also useful for finding maximum amplitude of oscillating systems or their maximum velocities.
7. Energy diagrams can show you whether or not a situation is stable. This can also be used in oscillations, as described below.

Problem class five-- oscillations

1. The most straightforward way to solve for the frequency of an oscillation is from newton's laws.
2. Write newton's law. $F=ma$ (or $T=I\alpha$).
3. Put it in the form $\frac{d^2x}{dt^2}+\omega^2x=0$
4. The frequency is read off from the equation in this form. To prove this involves differential equations.
5. Alternatively, you can use energy considerations to find the frequency.
    1. Write a formula for the total energy at an arbitrary point.
    2. Manipulate the equation so that it is of the form $\frac{1}{2}mv^2+\frac{1}{2}kx^2=constant=E$
    3. $\omega=\sqrt{\frac{k}{m}}$
6. In order to simply calculate the spring constant, you can most easily use energy.
    1. Write a formula for the total energy at an arbitrary point.
    2. By using a taylor-series expansion, we can approximate the energy around a point of minimum energy to be a parabola.
    3. Thus, you can approximate $E=\frac{1}{2}k_{eff}x^2$ and differentiate twice with respect to x to get k_{eff} as a function of x. Procedurally, this means that you take the second derivative of your energy function and then plug in x_0
    4. Plug in your equilibrium x_0 to get k_{eff} around that point

Problem class six-- rotation:

1. Statics are problems where you have a non-point object that is not moving and you are asked to calculate forces, for example.
    1. Draw a picture, clearly labeling all the forces on the system.

2. Use that the sum of forces in the x and y directions are both zero to find two equations.
3. Use that the sum of torques is zero to find a third equation.
4. Solve the equations.

2. Dynamics are problems involving movement.
   1. Write an equation from $T = I\alpha$ Solve equation for α. Finis.
   2. $I \equiv \int r^2 dm$ over your body with r=0 as your origin. Generally, to do this integral, you put dm in terms of dr.

Problem class seven-- angular momentum and torque:

1. Every rotating system can be represented as a translational part and a rotation around the center of mass (for more info, see page 274-- this also happens to give a good example for where reduced mass comes from)
2. The angular momentum is a radius vector from a (stationary) origin in your frame of reference crossed with the linear momentum. Make sure your origin is stationary. If it's not you can run into some serious trouble unless you're very careful.
3. This can be simplified to $L = r_{cm} \times P_{translational} + I\omega$. In this equation, your origin of rotation is the center of mass for calculation of both I and ω. Alternatively, you can make your origin something else as long as your origin is inertial.
4. The change in angular momentum is equal to the integral of the net torque with respect to time. $dL = T\,dt$
5. To solve problems involving collisions of spinning objects, you use the exact same method you use to solve collisions of translating objects, except you use $L(t+\Delta t) - L(t) = T\Delta t$. For continuous collisions, make $\Delta t \rightarrow dt$ and find the angular acceleration as you found translational acceleration before.

Problem class eight-- angular kinetic energy:

1. Angular kinetic energy is frequently written as $\frac{1}{2} I\omega^2$
2. Now you can use this energy in the same way that you used other energies above. The translational kinetic energy is independent of the angular kinetic energy.
3. If you are solving problems involving rolling bodies, it is useful to use the relation that $\omega = \frac{v}{r}$ to correlate translational and angular kinetic energies.

Problem class nine-- Fictitious Forces(p. 343, 355)

1. By using Einstein's principle of equivalence, you can show that any sufficiently small accelerating system can be equivalently drawn as a stationary system in a gravitational

field.

2. Essentially, this principle says that if a system is accelerating with an acceleration vector $\vec{a}$ that system can be viewed totally equivalently as a stationary system feeling an external force equal to $-m\vec{a}$
3. For a mass accelerating as a rate a with respect to inertial coordinates and at a rate a_{rot} with respect to a rotating coordinate system, then the equation of motion for the system is F=ma.
4. In rotational coordinates, $\vec{F_{rot}} = m\vec{a_{rot}}$
5. $\vec{a} = \vec{a_{rot}} + \vec{A}$, where A is the relative acceleration.
6. Therefore, $\vec{F_{rot}} = m(\vec{a} - \vec{A}) = \vec{F} + \vec{F_{fict}}$
7. Therefore, $\vec{F} = m\vec{a}, \vec{F_{fict}} = -m\vec{A}$
8. For circular motion, $\vec{a_{rot}} = \vec{a_{inertial}} - 2\vec{\omega} \times \vec{v_{rot}} - \vec{\omega} \times (\vec{\omega} \times \vec{r})$ (derivation on page 355-359)
9. Finally, the $\vec{F_{fict}} = -2m\vec{\omega} \times \vec{v_{rot}} - m\vec{\omega} \times (\vec{\omega} \times \vec{r})$

Problem class ten-- Central Force problems:

1. The reduced mass is $\mu = \frac{m_1 m_2}{m_1 + m_2}$ (derived on page 378-379) This represents the center of mass of the system and takes into account the reduced radius to the center of mass as compared to the other object.
2. Energy and angular momentum are constant. You can use these to calculate most things about such systems.
3. The total energy of a system like this is $\frac{1}{2}\mu v^2 + U(r)$
4. If you split up v into a radial and angular component, you get: $\frac{1}{2}\mu[(\frac{dr}{dt})^2 + r^2(\frac{d\theta}{dt})^2] + U(r)$
5. Conservation of angular momentum allows you to replace $\frac{d\theta}{dt} \Rightarrow \frac{L}{\mu r^2}$
6. Therefore, the new (equivalent) energy equation is: $\frac{1}{2}\mu(\frac{dr}{dt})^2 + \frac{1}{2}\frac{L^2}{\mu r^2} + U(r)$
7. You can then define the effective potential energy to be $\frac{1}{2}\frac{L^2}{\mu r^2} + U(r)$

8. The equation for energy is now $\frac{1}{2}\mu\left(\frac{dr}{dt}\right)^2 + U_{eff} = E$ (the above derivation is on page 381)
9. The graph of U_{eff} will tell you about the orbits
10. If you differentiate U_{eff} as discussed in the section on oscillations twice, you will get an approximate value for k_{eff}. You can use that $\omega = \sqrt{\frac{k}{m}}$ to find how particles in a potential field behave when perturbed slightly from equilibrium.
11. For information on orbits, see pages 390-406

Testing advice:

1. Sleep well before a hard exam. Don't stress out about it.
2. Warm up before the test begins. Start thinking about physics problems before you start the test and when you see the first problem on the final, you won't do what I did-- stare at it for 20 minutes trying to remember how to work with momentum. Even though it means waking up earlier, it's very worth it.